

Quasiparticle transitions in a charge-phase qubit probed by rf-oscillations

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We investigated transitions in an Al charge-phase qubit of SQUID-configuration which was inductively coupled to an rf tank circuit that made it possible to read out the state by measuring the Josephson inductance of the qubit. Depending on the flux and charge bias and on the amplitude of the rf-oscillations, we probed either the ground state or a dynamic change of the qubit states which we attributed to stochastic single quasiparticle tunneling onto and off the qubit island, involving an exchange of energy with the qubit. Within the scope of this model, a selection rule for quasiparticle-induced transitions in the qubit is discussed.

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Superconducting circuits based on small Josephson junctions and enabling the tunneling of single Cooper pairs offer great opportunities for electronics and quantum computing. The class of the single Cooper pair devices comprises Bloch transistors [1, 2, 3], quantum electrometers [4, 5, 6], $2e$ -pumps [7], etc. A prerequisite for the regular operation of these devices is that no quasiparticle (QP) transitions occur and that the so-called even-parity state of the small superconducting island is thus maintained. If this requirement is not fulfilled, even a single QP can instantly change the island charge, leading to a shifting of the operation point.

There are several ways of observing the even-odd states of the Bloch transistor island experimentally. These include: (i) the measuring of the gate dependence of the Josephson switching current (see, e.g., Ref.[3]), (ii) the monitoring of the supercurrent peak [2, 8], and (iii) the measuring of the island charge q in the zero-current-biased transistor (i.e. Cooper pair box) by means of a capacitively coupled SET electrometer [9, 10, 11, 12]. Recently, Naaman and Aumentado [13] have investigated the parity states of a current-biased Bloch transistor included in a resonance LC -circuit driven by a 500 MHz harmonic signal. In this circuit, the charge-dependent parameter was the Josephson inductance $L_J(q)$ of the Bloch transistor in the ground state.

In this paper, we will address the problem of single QPs tunneling in Josephson qubit circuits. The operation of charge qubits [14] is, by nature, sensitive to the incoherent tunneling of unpaired electrons because they change the charge state of the system instantaneously and stochastically [15, 16]. This makes the setting of the optimal working point of the qubit difficult and increases the decoherence [16, 17].

Our circuit comprises a Bloch transistor which is included in a superconducting ring inductively coupled to an rf-driven tank circuit [17, 18, 19, 20]. In such an rf-SQUID configuration, the Bloch transistor is galvanically decoupled from the measuring circuit, which in general leads to a reduction in the density of non-equilibrium QPs that are able to enter the island. The effect of the QP transitions manifested itself in excitations of this qubit and was detected by measuring the change in the Joseph-

son inductance value $L_J(q)$. We will demonstrate that in the general survey, the dynamics of our circuit can be described in a similar way as in the model of Lutchyn *et al.* [21], which has been published recently and addresses the problem of the energy relaxation in the charge qubit caused by a single QP. In contrast to Ref.[21], however, we will focus in this paper on the reverse process, i.e. on the energy transfer from the QP to the qubit, leading to an excitation of the qubit. Moreover, we will derive a selection rule showing that the rates of the QP transitions strongly depend on the initial qubit state.

In our Al qubit circuit (see Fig.1 (a)), the superconducting loop which closes the Bloch transistor has a small geometrical inductance $L \ll L_J$. Thus, the total inductance of the closed loop is determined mostly by the transistor's Josephson inductance. The transistor is operated as a Cooper pair box (qubit) whose distinct quantum states - which have the ground state energy E_0 and the excited state energy E_1 (see Fig. 1(b)) - are associated with different Bloch-bands of the system [22]. The quantum states of the transistor also depend on the phase coordinate φ_{dc} determined by the external magnetic flux Φ_{dc} that is applied to the loop $\varphi_{dc} = 2\pi\Phi_{dc}/\Phi_0$, with Φ_0 being the flux quantum. The Josephson inductance L_J is related to the local curvature of the corresponding energy surface $E_n(q, \varphi)$, i.e. $1/L_J(n, q, \varphi) \propto \partial^2 E_n(q, \varphi)/\partial \varphi^2$, $n = 0$ and 1 . Thus, the qubit eigenstates can be identified by means of the Josephson inductance of the transistor which is probed by small rf-oscillations in the loop, with the resulting phase $\varphi(t) = \varphi_{dc} + a \sin(2\pi ft)$ and with a being proportional to the amplitude of the rf-oscillations in the tank circuit induced by an rf-driving current. The black arcs on the energy surfaces in Fig. 1 (b) illustrate the rf-oscillations of the Josephson phase, which in turn, show the curvature of the band profile and are detected by the tank circuit coupled to the qubit. The drive frequency f is close to the bare resonance frequency $f_0 \approx 77$ MHz of our Nb-tank circuit which has a quality factor of $Q \approx 370$. The symmetric double loop is coupled - through the mutual inductance M - to the coil of the double spiral tank circuit formed as a planar gradiometer. The coupling coefficient is $k = M/\sqrt{LL_T} \approx 0.4$, with $M \approx 3.8$ nH, $L \approx 0.7$ nH

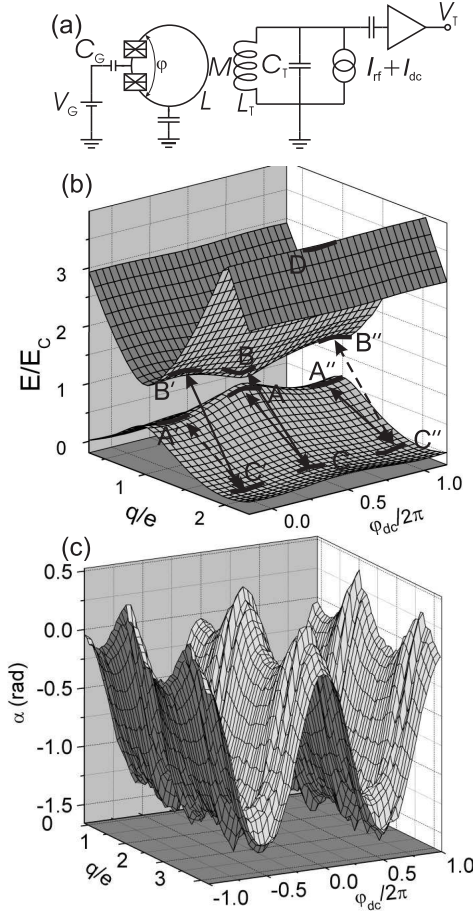


FIG. 1: (a) Diagram of the measurement set-up. The core element is a double Josephson junction (the two crossed boxes) with a capacitive gate coupled to its island, i.e. the Bloch transistor, embedded in a macroscopic superconducting loop. The loop is inductively coupled to an rf-driven tank circuit which is capacitively coupled to a cold preamplifier. (b) Energy band diagram of the circuit, calculated for the parameters found in the experiment. The thick-line arcs show the variations of the phase φ caused by oscillations in the tank circuit. The arrows denote the quasiparticle-induced transition between the qubit states (the dashed arrows indicate transitions suppressed due to the destructive interference effect, see text below). (c) Phase shift α in the tank circuit, measured as a function of the dc-value of the Josephson phase φ_{dc} and the island charge q for an amplitude $a = 0.56$ rad of rf-oscillations of the phase.

and the tank circuit inductance $L_T \approx 150$ nH (see Ref. [20], where the behavior of an all-Nb circuit of similar design was investigated). Due to the coupling to the qubit, the effective inductance L_{eff} of the resonance circuit is equal to $L_T - M^2 L_J^{-1}(n, q, \varphi)$.

The Bloch transistor, the loop and the tank circuit were fabricated by electron beam lithography on the same chip. The tank circuit inductor was fabricated on the basis of Nb technology [20, 23], the qubit loop and the Bloch transistor by means of the two-angle Al shadow evaporation technique. No special precautions for the

suppression of QP-poisoning of the island - such as, for example, the engineering of a barrier-like gap profile with the island gap value being greater than the electrode gap value [24, 25], or the implementation of normal-metal QP traps [3] in the outer electrodes - were taken. The critical currents of the single junctions were approx. 25 nA, with the corresponding value of $45 \mu\text{eV}$ for the average Josephson coupling energy $E_{J0} = (E_{J1} + E_{J2})/2$ ($\approx E_{J1} \approx E_{J2}$, whereby the corresponding values of the single junctions of the transistor yield the effective Josephson coupling energy $E_J(\varphi) = \sqrt{E_{J1}^2 + E_{J2}^2 + 2E_{J1}E_{J2}\cos\varphi_{dc}}$). The charging energy E_C of the transistor island has a value of $110 \mu\text{eV}$, so that both energies E_C and E_{J0} are smaller than the value of the superconducting gap in Al films, $\Delta_{Al} \approx 210 \mu\text{eV}$. These values of E_{J0} and E_C were taken from a fitting of the shape of the ground state extracted from rf-measurements with finite amplitude of the Josephson phase oscillations, see Ref.[20] for details. Moreover, the obtained data agreed well with the corresponding Ambegaokar-Baratoff and Coulomb-blockade values of similar test transistors, extracted from dc-measurements of their I-V characteristics.

Unfortunately, due to the technical conditions (poor signal-to-noise ratio for a small amplitude of the phase oscillations), it was not possible to determine the Josephson energy difference $\Delta E_J = |E_{J1} - E_{J2}| \equiv E_J(\varphi_{dc} = \pi) \ll E_{J0}$. An analysis of the fitted curves $\alpha(q)$ shows that their shape for the smallest amplitude $a = 0.28$ of the phase oscillations does not depend on the asymmetry factor $\Delta E_J/E_{J0}$ if this ratio is smaller than 0.2. From this we conclude that the asymmetry factor in our sample is in the range $\Delta E_J/E_{J0} \leq 0.2$.

In our experiment, which was carried out in a dilution refrigerator at a base temperature of 20 mK, we measured the phase angle α between the driving signal I_{rf} and the rf voltage V_{rf} on the tank circuit. From the α -dependence one can deduce the Josephson-inductance $L_J(q, \varphi)$ by applying the simple formula

$$\tan \alpha = k^2 Q \frac{L}{L_J(q, \varphi)} \quad (1)$$

which applies when a drive frequency f is equal to the bare resonance frequency f_0 . The measurement of the phase shift, α , for different values of the dc phase and the gate charge allows the curvature of the energy surface to be mapped. Figure 1 (c) shows a periodical dependence of α both on $\varphi_{dc} = (2\pi/\Phi_0)MI_{dc}$ and $q = C_G V_G + q_{offset}$, where we find a $2e$ -periodic gate modulation. As had been expected for the ground-state shape, a plain gate modulation dependence with a peak-to-peak amplitude of 0.6 rad is to be seen in Fig.1 (c) for $\varphi_{dc} = 0$. As can be expected from the theoretical dependence of the ground state energy, the gate modulation curve for $\varphi_{dc} = \pi$ is inverted with respect to the $\varphi_{dc} = 0$ -curve, which is expressed by the opposite sign of the energy surface curvature. A closer look at the gate-dependence curve measured at $\varphi_{dc} = \pi$ reveals, however, a peculiar shape. Before reaching the zenith of the curve, the phase angle

α starts to rise sharply and remains in a broad range around $q = e$ at a level that is even higher than that for $q = 0$ (this dependence is also shown in Fig. 2(a) by symbols).

Our theoretical understanding of this behavior bases on the assumption of a statistical mixture of the different quantum states of the qubit. Here, at the degeneracy point ($q = e$ and $\varphi_{dc} = \pi$), the states considered are the ground state A and the excited state B , both with even parity of the island, and the ground state C and the excited state D , both with odd parity, as shown by the black arcs in Fig. 1 (b). In our opinion, the excited odd-parity state D does not play any significant role, as its excitation energy is too great (i.e. around $3E_C$) and hence much larger than the energy gap ΔE_J between A and B . We can also rule out a notable contribution of the ground state C with odd parity, as the admixture of the values of $\alpha(q)$ corresponding to this state (indicated by the corresponding black arcs in Fig. 1 (b)) with small negative curvature cannot yield the above-mentioned overshooting of the experimental data in the vicinity of $q = e$. Likewise, we are of the opinion that the contribution of the odd state is small due to the presumably short lifetime of a QP in the island, see, e.g., Ref.[21, 24]. Therefore, we modelled the experimental data as a mixture of the ground state A and the excited state B , with the signs opposite to each other. This was done by replacing, in Eq. (1), the Josephson inductance with

$$L_J^{-1}(q) \rightarrow w_0(q)L_J^{-1}(q)|_{n=0} + w_1(q)L_J^{-1}(q)|_{n=1} \quad (2)$$

(whereby the weight factors are non-negative occupation numbers which obey the relation $w_0(q) + w_1(q) = 1$) and have been fitted subsequently to the measured $\alpha(q)$ -dependence, see Fig. 2 (a). In this way, we were able to reconstruct the occupation numbers $w_{0,1}(q)$ of the ground state and of the excited state, see Fig. 2 (b). As a result, we found at $q = \pm e$ an increase in the occupation of the upper state up to $w_1(e) = 0.46$, which remained rather large in a broad range around this degeneracy point.

The found steady-state populations $w_{0,1}$ yield the ensemble-averaged values of the Josephson inductance according to Eq. (2). The ratio of w_1 to w_0 reflects the competition between the rates of the QP tunneling and the energy relaxation of the qubit. Such a relaxation also occurs due to the coupling to the environmental degrees of freedom (e.g. flux and gate control lines, external magnetic field, background charge, etc.). From our measurements at $\varphi_{dc} = \pi$ we can conclude that the relaxation due to the environmental degrees of freedom which reduces w_1 and increases w_0 is not dominant, because in our case $w_0 \approx w_1 \approx 0.5$. Since we measure with our technique only averaged values of the phase shift α , we are not able to find absolute values of the relaxation rates. Beyond this, an accurate determination of the different relaxation rates is only possible when the distribution of the non-equilibrium QPs is known with regard to their energy. To explain this mixture effect, we rule out a thermal activation of the excited state at the given

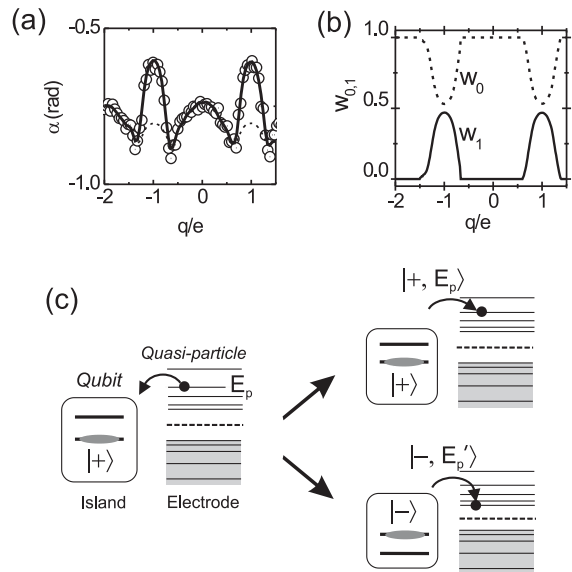


FIG. 2: (a) Comparison between the experimental (symbols) and the calculated (solid line) gate modulation dependencies $\alpha(q)$ for an amplitude of rf-oscillations of $a = 0.56$ rad and a phase bias $\varphi_{dc} = \pi$. (b) Reconstructed occupation probabilities of the ground state (dotted line) and of the excited state (solid line), yielding the modelled dependence in (a). (c) QP-pumping mechanism: in the left panel, an unpaired non-equilibrium QP tunnels from the outer electrode onto the island of the qubit in the ground state. In the favoured process, the QP tunnels - due to the somewhat larger density of states there - back into a lower-energetic state of the outer electrode and, hence, transfers energy to the qubit by exciting it to the upper state (see panel at the lower right). The alternative process, i.e. the tunneling of a QP back to the outer electrode without the qubit being excited is shown in the panel at the upper right.

base temperature, which is below 100 mK, because at such low temperatures such high w_1 -values can never be achieved, i.e. the Boltzmann-factor for the excited state is negligible. Besides, the broad range of the gate charge where mixture occurs (more than 40% of the gate charge period) cannot be achieved by such a weak thermal excitation. This is shown by the following estimation: for a finite detuning from the degeneracy point $q = e$, the gap energy increases roughly like $\Delta E = 4E_C|q/e - 1|$ and the factor $\exp(-\Delta E/k_B T)$ is therefore strongly suppressed at a finite detuning $\delta q \equiv q/e - 1 = 0.1$ for $E_C \approx 110 \mu\text{eV}$ and $T = 100$ mK.

Another possible explanation for the observed mixture effect could be Landau-Zener (LZ) tunneling due to the rf-drive which leads to a periodic passing of the degeneracy point (at $\varphi_{dc} = \pi$ and $q = e$). When we take the broad range of the mixture effect into account, LZ tunneling seems to be unlikely, since it is exponentially suppressed for a finite δq . When we apply the parameters of our experiment, the estimated LZ tunneling probability

p_{LZ} is negligible already for small values of $\delta q = 0.1$:

$$p_{\text{LZ}} = \exp \left[-\pi \frac{\Delta E^2}{2\hbar \dot{E}} \right] = \exp \left[-\frac{\Delta E^2}{4a\hbar f E_{J0}} \right] \ll 1 \quad (3)$$

for $E_{J0} \approx 45 \mu\text{eV}$, $E_C \approx 110 \mu\text{eV}$ and an intermediate amplitude $a = 0.5$ of rf-oscillations. On the other hand, experimentally we still observe a strong contribution of the excited band at $\delta q = 0.1$.

A further argument against LZ tunneling is the behavior of the curves measured at $\varphi_{\text{dc}} = 0$ and at values of q around e (except for those at $q = e$, which will be discussed later). They clearly show the effect of qubit excitation, whereas the qubit energy level spacing is, in this case, much larger than in the case of $\varphi_{\text{dc}} = \pi$.

Following a model given by Ref.[21], we relate the observed mixed state to a strong coupling of non-equilibrium QPs to the qubit system, which allows the transfer of energy to the qubit and thus an excitation of its upper state. This QP pumping of the qubit can be considered as a cycle in which an unpaired non-equilibrium QP tunnels onto the island while the qubit is in the ground state $|+\rangle$, see Fig.2 (c). As soon as the QP enters the transistor island, it changes the excess charge and induces in this way an instantaneous change of the working point and of the energy level splitting. Only when the QP tunnels to a lower-energetic state of the electrode, a transfer of energy to the qubit system occurs, exciting it to the upper state $|-\rangle$, whereas a QP tunneling back to the initial state of the electrode does not induce any excitation of the qubit. According to Ref. [21], the former process should prevail due to the greater density of the states that are close to the edge of the QP band in the energy spectrum [26]. Of course, such non-equilibrium QPs should have an energy which is at least by the value of E_J larger than the value of the superconductor energy gap, in order to transfer energy to the qubit. This would mean, however, that a QP having a lower energy could enter the island as well, but in that case the QP should leave the island without exciting the qubit. Trapping the QP in the island is only possible if the superconductor energy gap of the island is lower than that of the electrodes. Due to particularities of our sample fabrication, the energy gaps of the electrodes and of the transistor island are almost equal. Moreover, we presume that the gap of the island is even slightly larger. We assume that QPs having an energy lower than $\approx E_J$ above the energy gap are available in the outer electrodes and that their relaxation to the gap edge occurs both via interaction with the lattice of the electrodes and via the traveling onto the island and back into the electrodes with simultaneous excitation of the qubit.

The pumping process depends strongly on the qubit bias and on the amplitude of the rf-oscillations. This is demonstrated in the gate modulation curves $\alpha(V_G)$ in Fig.3 for different values of the dc phase bias. Interestingly, when increasing the amplitude of the rf-oscillations for $\varphi_{\text{dc}} = 0$ (top part of the plot) from 0.28 rad to 1.12 rad, two dips appear around the sharp peak at $q = e$.

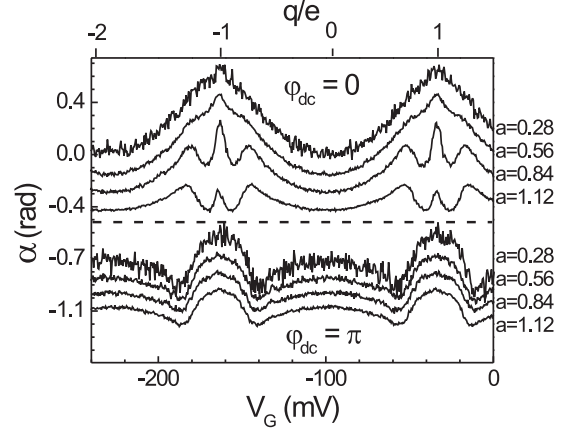


FIG. 3: Periodic gate modulation curves $\alpha(V_G)$ for different values of the amplitude of the rf-oscillations, measured at two values of the dc phase bias. For the purpose of clarity, the curves are vertically shifted. The upper horizontal axis presents the values of the island charge.

This is in contrast to the behavior we have described for $\varphi_{\text{dc}} = \pi$ (bottom part of the plot). Here, the gate modulation curves practically do not depend on the amplitude a of the flux oscillations. We interpret these observations in the following way: for $\varphi_{\text{dc}} = 0$, the pumping process is not active for small rf-oscillations. It starts to develop at increasing values of a , and the shape of the curves shows a strong dependence of this mechanism on the charge q . In particular, the pumping is partially suppressed at $q = e$ even for rather large values of a . Therefore, we extend the theory of Ref.[21] by establishing a selection rule for the QP transitions between the ground state and the excited state for certain flux-bias-values.

The QP transitions are described by the tunneling Hamiltonian, H_T , which consists of two Hermitian conjugate terms and is considered to be a perturbation term, see Ref. [21]. The Josephson coupling term describing the tunneling of Cooper pairs is naturally included in the Hamiltonian of the qubit. To describe this peculiar behavior observed in the experiment, we should take the interference effect into account which occurs when QPs tunnel onto the island. Let us assume that the energies of the non-equilibrium QP in the electrode $E_p = (\Delta^2 + \xi_p^2)^{1/2}$ and in the island $E_k = (\Delta^2 + \xi_k^2)^{1/2}$ exceed the gap value Δ only slightly, i.e. the corresponding electron energies are sufficiently small, $|\xi_p|, |\xi_k| \ll \Delta$. In this case the absolute values of the Bogoliubov-Valatin coherence factors of the corresponding single QP states [27] are almost equal to each other,

$$u_p^2 \approx v_p^2 \approx u_k^2 \approx v_k^2 \approx 0.5. \quad (4)$$

This means that QPs can tunnel onto the island both as an electron-like particle and as a hole-like particle. The qubit biased by the gate charge $q = (N + 1)e$ with $N = 0, \pm 2, \dots$ switches into the odd (ground) state $|o\rangle$ following two paths, $|\pm\rangle \rightarrow |N\rangle$ and $|\pm\rangle \rightarrow |N + 2\rangle$,

as shown in Fig. 4 (a) by arrows. The sign "+" ("−") stands for the even ground state (excited state) of the qubit. These states represent coherent superpositions of states with a different number of Cooper pairs on the island (see, e.g., Eq. (5) in Ref.[17]). These phase factors in the series do not only depend on the particular state of the qubit, but also on the value of the magnetic flux Φ which penetrates the qubit loop and thus determines the overall Josephson phase $\varphi = \varphi_1 + \varphi_2$. This dependence is expressed most simply in the case of the small ratio E_J/E_c , i.e.,

$$|\pm\rangle = (|N\rangle \pm e^{i\varphi/2}|N+2\rangle)/\sqrt{2}, \quad (5)$$

while in the general case a low-weight admixture of higher order terms with corresponding signs and phases, i.e., $(\pm 1)^k e^{ik\varphi/2}|N+2k\rangle$, where integer $k = -1, -2, -3, \dots$ and $k = 2, 3, 4, \dots$, should also be added. One can see that in contrast to the case of a plain Cooper pair box which, in the degeneracy point, has a symmetric (antisymmetric) wave function in the ground (excited) state, the symmetry property of the states $|\pm\rangle$ in the box of the SQUID configuration alternates with the flux value Φ . Especially, for values which correspond to an even number of flux quanta, $\Phi = m\Phi_0$, $m = 0, \pm 2, \dots$, yielding $e^{i\varphi/2} = 1$, the ground state (excited state) of the qubit is described by a symmetric (antisymmetric) wave function, whereas for odd numbers of flux quanta, $m = \pm 1, \pm 3, \dots$, the ground state (excited state) is anti-symmetric (symmetric) [28]. Therefore, at any integer value of flux quanta in the loop, at least one of the states (ground state or excited state) is necessarily symmetric at $q = e$. As we see below, this property plays an important role for establishing the selection rule for the rate of the QP tunneling which includes these states.

The operators causing transitions in the qubit are the charge shift operators $e^{\pm i\phi/2}$, where $\phi = (\varphi_1 - \varphi_2)/2$ is the variable conjugate to the island charge operator (see, e.g., Ref.[29]). These terms appear in the tunneling Hamiltonian terms as phase factors of the creation- and annihilation-operators associated with the island. Physically, these phase factors ensure a coupling between the tunneling particle and the environment, i.e. the charge degree of freedom of the qubit, cf. Ref.[29]. Applying the Fermi Golden rule for calculating the transition rates of the QP tunneling onto the island, we obtain

$$\Gamma_{\pm}^{\text{in}}(E_p, E_k) = 2\pi |\langle E_p, \pm | H_T | o, E_k \rangle|^2 \delta(E_p + E_{\pm} - E_k). \quad (6)$$

Taking into account the simplifying assumption Eq. (4), the matrix elements in Eq. (6) can be presented as

$$\begin{aligned} |\langle E_p, \pm | H_T | o, E_k \rangle|^2 &= |t_{pk}|^2 [|e_{\pm}|^2 |u_p|^2 |u_k|^2 + |e'_{\pm}|^2 |v_p|^2 |v_k|^2 \\ &\quad - e_{\pm} e'_{\pm} u_p u_k v_p v_k - e'_{\pm} e_{\pm} u_p u_k v_p v_k] \\ &= |t|^2 |e_{\pm} - e'_{\pm}|^2 / 4, \end{aligned} \quad (7)$$

where the matrix elements are $e_{\pm} = \langle \pm | e^{i\phi/2} | N \rangle$ and $e'_{\pm} = \langle \pm | e^{-i\phi/2} | N+2 \rangle$. Here $|t_{pk}|^2 \equiv |t|^2$ is the square

of the absolute value of the tunnel matrix element which is only weakly dependent on the energies E_p and E_k . Therefore, the net rate of the QPs tunneling onto the island can be expressed as $\Gamma_{\pm}^{\text{in}} \propto (\delta_r/R_T)W_{\pm}$, with $W_{\pm} = |e_{\pm} - e'_{\pm}|^2$ (see the plots in Fig. 4 (b)) and δ_r being the QP level spacing in the island.

Since at $q = e$ one of the qubit states $|+\rangle$ (for even m) or $|-\rangle$ (for odd m) is described by a symmetric wave function, the matrix elements e_+ and e'_+ (for even m) or e_- and e'_- (for odd m) are identical, which leads to zero value for W_+ or W_- . This manifests itself in the zero minimums in the two lower curves in Fig. 4 (b), as well as in the suppression of the corresponding transition rates Γ_+^{in} or Γ_-^{in} , respectively. In contrast to this, the transitions in which asymmetric qubit states are involved have an increased probability (the two upper curves in Fig. 4 (b)) due to the constructive interference which results from the opposite signs of the corresponding matrix elements. The effect of the complete destructive interference occurring in transitions involving the symmetric states of the qubit is formulated as a selection rule. Note that the same selection rule is applied to transitions which are associated with the tunneling of the QP from the island back into the electrode with simultaneous switching of the qubit into a state with symmetric wave function. If finite QP energies ξ_q and ξ_k above the superconductor gap are taken into account, this leads to a somewhat unbalanced ratio of the coherence factors u and v of Eq. (4) and, hence, to an incomplete destructive interference by which the selection rule is violated.

The destructive interference described is supposed to have a suppressing effect on the cyclic qubit excitation. In our scenario of the qubit pumping at $\varphi_{\text{dc}} = \pi$, the rates of both intraband and interband transitions were assumed to be much larger than the rate γ_r of the qubit relaxation $B \rightarrow A$. In this case the pumping cycle is closed and the two qubit states A and B are almost equally populated at $q = e$, see Fig. 2 (b). In this case, the average phase angle α is positioned close to, but slightly higher than the level of the state C , manifesting itself in the experimental $\alpha - V_G$ -curve in Fig. 2 (a) as an overshooting around $q = e$. This process is illustrated by the time-trace of repeated qubit transitions in the bottom panel of Fig. 4 (c) for $\varphi_{\text{dc}} = \pi$, i.e. for the case where neither of the states is symmetric. Let us consider now the phase bias $\varphi_{\text{dc}} = 0$, where - as has been discussed before - the qubit ground-state has a symmetric wave function with a corresponding low rate γ_{symm} for transitions into this state. Due to the effect of the destructive interference (denoted by the dashed arrows in Fig. 1 (b) and Fig. 4 (a)), the rate of intraband transitions $A' \rightleftharpoons C'$ is presumably much slower than the qubit relaxation. As a result, the qubit remains in the even symmetric state A' for a sufficiently long time or switches fast between the even antisymmetric state B' and the odd state C' on an average time scale γ_r^{-1} (see the time-trace for $\varphi_{\text{dc}} = 0$ in the central panel of Fig. 4 (c)). If, on the other hand, the flux bias is odd, e.g. $\varphi_{\text{dc}} = 2\pi$, the excited state of the

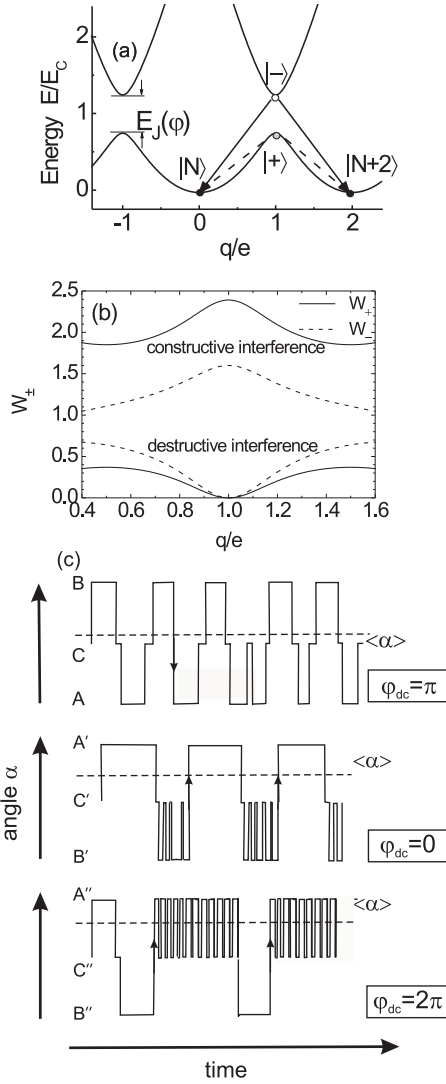


FIG. 4: (a) Energy diagram of the qubit, displaying the interfering paths of the excess QPs tunneling either - as a quasi-electron (to the left) - or - as a quasi-hole (to the right). (b) The rate factors for the intraband transition probability W_+ and interband transition probability W_- , calculated for the experimental parameters of the circuit for the integer number m of flux quanta in the loop as a function of the gate charge. For even m , the destructive (constructive) interference takes place for the intraband (interband) transitions, and for odd m vice versa. (c) Schematic time-traces of the cyclic transitions between qubit states at $q = e$ for different values of φ_{dc} shown in Fig. 1 (b). The thin arrows indicate the qubit relaxation in contrast to the quasiparticle-induced transitions.

qubit is symmetric, which leads to a strong reduction in the interband transitions $B'' \rightleftharpoons C''$. The schematic time-trace in the central panel in Fig. 4 (c) shows for this case a long series of rapid transitions $A'' \rightleftharpoons C''$, interrupted by rare (with a rate of $\sim \gamma_{\text{symm}}$) transitions into the state B'' . The escape from the excited state B'' with an average occupation time γ_r^{-1} mainly occurs via qubit relaxation $B'' \rightarrow A''$. When we summarize the case of

an integer dc phase bias $\varphi_{dc} = 2\pi m$, the sufficiently high rate of relaxation with respect to the small rate due to the destructive interference ($\gamma_r \gg \gamma_{\text{symm}}$) at $q = e$ ensures that $\langle \alpha \rangle$ is close to the value corresponding to the ground state A' or A'' . Hence, this average value of the phase angle shows that the destructive interference effect suppresses the pumping cycle.

This interference effect can qualitatively explain the shape of the $\alpha - V_g$ dependencies measured at zero flux value and shown in the upper part of Fig. 3. The two upper curves, measured at a small amplitude a of rf-oscillations, exhibit a shape which corresponds almost entirely to the expected form of the ground state. This behavior is related, first of all, to relatively small deviations of the Josephson φ from the zero value, in the vicinity of which the destructive interference is strongest and, secondly, to a relatively large QP energy ($\sim 2E_{J0}$) which is required for the excitation of the qubit. The increase in the amplitude a leads to distinct deviations of the Josephson phase φ from the zero value, so the destructive interference is still efficient only in a narrow region around the value $q = e$. This effect manifests itself in a sharp peak centered around the value $q = e$, whereas slight deviations from this value lead to the two side dips and are an indication of a significant admixture of the excited state. The described behavior can be understood as a significant weakening of the destructive part of the interference, which is due to the deviation from the optimum point both in the dc phase and charge. Moreover, the excitation qubit energy is in this case somewhat lower than $\sim 2E_{J0}$. Note that in the case of a semi-integer flux quantum bias (the respective curves are presented in the bottom part of Fig. 3 for $\varphi_{dc} = \pi$), the effect of destructive interference does not show. This is understandable, because neither of the wave functions describing the ground state and the excited state possesses the property of symmetry (see Eq. (5) in which the parameter is $\varphi = \pi$).

In conclusion, we found the effect of mixing of the qubit states, which we explained by an energy transfer between the non-equilibrium quasiparticles and the two-level qubit system. To explain the qubit excitation observed in the experiment, we applied a picture of the stochastic tunneling of single quasiparticles and derived a selection rule for the quasiparticle tunneling at the magic points $q = e$ of the qubit. This selection rule is applicable to transitions in which qubit states with a symmetric wave function are involved, and it strongly suppresses the mechanism of the qubit pumping. Our set-up only permitted a continuous readout of the - therefore - averaged qubit state. A pulsed readout set-up with a higher drive frequency could help to discriminate the single tunneling events and allow probing the ground state and the excited state of the qubit separately.

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- [1] D. V. Averin and K. K. Likharev, in *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, 1991), p.175.
 - [2] M. T. Tuominen, *et al.*, Phys. Rev. Lett. **69**, 1997 (1992).
 - [3] P. Joyez, *et al.*, Phys. Rev. Lett. **72**, 2458 (1994).
 - [4] A. B. Zorin, Phys. Rev. Lett. **76**, 4408 (1996).
 - [5] A. B. Zorin, *et al.*, J. Supercond. **12**, 747 (1999); A. B. Zorin, Phys. Rev. Lett. **86**, 3388 (2001).
 - [6] M. A. Sillanpää, L. Roschier and P. Hakonen, Phys. Rev. Lett. **93**, 066805 (2004).
 - [7] L. J. Geerligs, *et al.*, Z. Phys. B: Condens. Matter **85**, 349 (1991).
 - [8] A. Amar *et al.*, Phys. Rev. Lett. **72**, 3234 (1994).
 - [9] V. Bouchiat *et al.*, Phys. Scripta **T76**, 165 (1998).
 - [10] K. W. Lehnert *et al.*, Phys. Rev. Lett. **91**, 106801 (2003).
 - [11] J. Männik and J. E. Lukens, Phys. Rev. Lett. **92**, 057004 (2004).
 - [12] A. Guillaume *et al.*, Phys. Rev. B **69**, 132504 (2004).
 - [13] O. Naaman and J. Aumentado, Phys. Rev. B **73**, 172504 (2006).
 - [14] Y. Nakamura, Yu. A. Pashkin and J. S. Tsai, Nature **398**, 768 (1999).
 - [15] D. Vion, *et al.*, Science **296**, 886 (2002).
 - [16] T. Duty *et al.*, Phys. Rev. B **69**, 140503(R) (2004).
 - [17] A. B. Zorin, JETP **98**, 1250 (2004).
 - [18] A. B. Zorin, Physica C **368**, 284 (2002).
 - [19] D. Born *et al.*, Phys. Rev. B **70**, 180501(R) (2004).
 - [20] H. Zangerle *et al.*, Phys. Rev. B **73**, 224527 (2006).
 - [21] R. M. Lutchyn, L. I. Glazman, and A. I. Larkin, Phys. Rev. B **74**, 064515 (2006).
 - [22] K. K. Likharev and A. B. Zorin, J. Low Temp. Phys. **59**, 697 (1985); D. V. Averin, A. B. Zorin, and K. K. Likharev, Sov. Phys. JETP **88**, 697 (1985).
 - [23] R. Dolata *et al.*, J. Appl. Phys. **97**, 054501 (2005).
 - [24] J. Aumentado *et al.*, Phys. Rev. Lett. **92**, 066802 (2004).
 - [25] T. Yamamoto *et al.*, Appl. Phys. Lett. **88**, 212509 (2006).
 - [26] A finite rate of the qubit relaxation works opposite to the pumping direction.
 - [27] J. R. Schrieffer, *Theory of Superconductivity* (Perseus, Oxford, 1999).
 - [28] This property of the SQUID-configuration qubit was pointed out to the authors by D. V. Averin.
 - [29] G. L. Ingold and Yu. V. Nazarov, in *Single Charge Tunneling*, edited by H. Grabert, and M. H. Devoret (Plenum Press, New York, 1992), p. 21.
 - [30] Technically, these matrix elements were computed using the Fourier-series expansions of the Bloch-wave eigenstates of the qubit Hamiltonian, which were derived in Ref.[22].